



INDIAN SCHOOL AL WADI AL KABIR
HOLIDAY HOMEWORK 2014-15

Class: XII

Mathematics

06/06/15

1	Show that $\sin^{-1} \frac{12}{13} + \cos^{-1} \frac{4}{5} + \tan^{-1} \frac{63}{16} = \pi$.
2	Using properties of determinants, show that: $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$
3	If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, show that $(1-x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$
4	If $x = \sqrt{a \sin^{-1} t}$, $y = \sqrt{a \cos^{-1} t}$, show that $\frac{dy}{dx} = \frac{-y}{x}$.
5	Verify Rolle's Theorem for the following functions: $f(x) = (x-1)(x-2)^2$ on $[1, 2]$.
6	Evaluate: $\int \frac{dx}{(x+1)^2(x^2+1)}$
7	Evaluate: $\int \frac{x+2}{\sqrt{x^2+5x+6}} dx$
8	Obtain the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ using elementary operations.
9	Evaluate: $\int \sqrt{\tan x} dx$
10	Show that: $\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{2}{11} = \tan^{-1} \frac{3}{4}$
11	Construct a 2×2 matrix $A = [a_{ij}]$ if $a_{ij} = \frac{1}{2} i-3j $
12	If $A = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 0 \\ 3 & -1 \end{bmatrix}$, then find the matrix X. Such that $A+X=B$.
13	Find the values of x and y for which $\begin{bmatrix} 4 & x \\ y & 1 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 4 & 1 \end{bmatrix}$
14	Evaluate: $\int \sin^5 x dx$
15	Evaluate: $\int \frac{\cos x}{\sqrt{4+\cos^2 x}} dx$
16	Evaluate: $\int_0^{\frac{\pi}{4}} \log(1 + \tan x) dx$
17	Evaluate: $\int_0^4 (x + x-2 + x-4) dx$

18	For what value of x the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix
19	If A_{ij} is the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & 7 \end{vmatrix}$, then write the value of $a_{32}A_{32}$.
20	Write the anti derivative of $3\sqrt{x} + \frac{1}{\sqrt{x}}$.
21	Using principal values, find the value of $2\cos^{-1}\frac{1}{2} + 3\sin^{-1}\frac{1}{2}$.
22	Solve for x . $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \pi + \tan^{-1}(-7)$
23	Find the value of a for which the function f defined by $f(x) = \begin{cases} a \sin \frac{\pi}{2}(x+1), & x \leq 0 \\ \frac{\tan x - \sin x}{x^3}, & x > 0 \end{cases}$ is continuous at $x = 0$
24	Verify Rolle's Theorem for the function f given by $f(x) = e^x(\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.
25	Evaluate : $\int \frac{1}{\cos(x+a).\cos(x+b)} dx$
26	Evaluate $\int \frac{(2-x)e^x}{(1-x)^2} dx$
27	If $y = \log \sqrt{\frac{1+\tan x}{1-\tan x}}$, prove that $\frac{dy}{dx} = \sec 2x$
28	The management committee of a residential colony decided to award some of its members for honesty, some for helping others and some others for supervising the workers to keep the colony neat and clean. The sum of all awardees is 12. Three times the sum of awardees for cooperation and supervision added to two times the sum of awardees for honesty is 33. If the sum of the number of awardees for honesty and supervision is twice the number of awardees for helping others, using matrix method find the number of awardees of each category. Apart from these values suggest one more value which the management of the colony must include for awards
29	If $\log\left[\sqrt{1+x^2} - x\right] = y\sqrt{1+x^2}$ prove that $(1+x^2)\frac{dy}{dx} + xy + 1 = 0$
30	Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.
31	Evaluate $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$
32	Evaluate $\int (x-3)\sqrt{x^2+3x-18} dx$

33	For what value of x the matrix $\begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is a singular matrix.
34	Find x if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$
35	Evaluate: $\int \frac{\tan^4 \sqrt{x} \sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}} dx$
36	Find k if $f(x) = \begin{cases} kx^2 + 3, & x \leq 3 \\ 5x - 3, & x > 3 \end{cases}$ is continuous at $x = 3$
37	Solve for x : $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$
38	If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$
39	If $\sin y = x \sin(a+y)$, prove $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$
40	Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2}, & \text{if } x < 0 \\ 5x + 8, & \text{if } x = 0 \\ \frac{x}{\sqrt{16+x}-4}, & \text{if } x > 0 \end{cases}$. Prove $f(x)$ is continuous at $x=0$
41	If $A = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}$ then find $A^2 - 3A + 2I$
42	Evaluate: $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$
43	Evaluate: $\int \frac{(6\sin x + 7)\cos x}{\sqrt{(\sin x - 5)(\sin x - 4)}} dx$
44	Two institutions decided to award their employees for three values of resourcefulness, competence and determination in the form of prizes at the rate of Rs. x , Rs. y and Rs. z respectively per person. The first institution decided to award respectively 4, 3 and 2 employees with a total prize money of Rs 37000 and the second institution decided to award respectively 5, 3 and 4 employees with a total prize money of Rs. 47000. If all the three prizes per person together amount to Rs. 12000, then using matrix method find the values of x , y and z . What values are described in the question?
45	What is the value of the expression $\sin^{-1}\left(\frac{b}{\sqrt{a^2+b^2}}\right) + \sin^{-1}\left(\frac{a-b}{\sqrt{2(a^2+b^2)}}\right)$, $a > b$?