INDIAN SCHOOL AL WADI AL KABIR

Dept. of Mathematics

Class : XII HOLIDAY HOMEWORK Date: 06th June, 2016

1.	Express the matrix A as the sum of a symmetric and a skew symmetric matrix where A = $\begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$
2.	A= $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ $Prove: A^2 - 4A + 7I = 0. using\ this\ evaluate A^5$ Ans: $\begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$
3.	A= $\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, B= $\begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $find\ a\ matrix\ C\ such\ that\ 3A + 5B + 2C\ is\ a\ null\ matrix$ Ans: $\begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$
4.	If possible, using elementary row transformation, find the inverse of $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ Ans: $\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$
5.	A= $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, $find\ AB\ and\ hence\ solve:\ x - 2y = 10, 2x + y + 3z = 8\ and\ -2y + z = 7.$ Ans: x= 4, y= -3,z= 1
6.	Find the value of f($\frac{\pi}{4}$) so that f(x) becomes continuous at x = $\frac{\pi}{4}$, $where\ f(x) = \frac{\sqrt{2}cosx-1}{cotx-1}; x \neq \frac{\pi}{4}$. Ans: $\frac{1}{2}$
7.	Differentiate $tan^{-1}\frac{\sqrt{1-x^2}}{x}$ $with\ respect\ to\ cos^{-1}2x\sqrt{1-x^2},\ x \in \left[\frac{1}{\sqrt{2}}, 1\right]$ Ans: $-\frac{1}{2}$
8.	If $y^x = e^{y-x} prove\ \frac{dy}{dx} = \frac{(1+logy)^2}{logy}$
9.	If x = sint, and y = sinpt, prove $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$
10.	$If \int \frac{3e^x-5e^{-x}}{4e^x+5e^{-x}} dx = ax + log\|4e^x + 5e^{-x}\| + C, then\ find\ the\ values\ of\ a\ and\ b.$ Ans: a=- $\frac{1}{8}$, b= - $\frac{7}{8}$
11.	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{(a^2cos^2x+b^2sin^2x)^2}$ $Ans: \frac{\pi}{4}\left(\frac{a^2+b^2}{a^3b^3}\right)$
12.	$Evaluate: \int \frac{cos2x - cos2\theta}{cosx - cos\theta} dx$ $Ans: 2(sinx + xcos\theta) + C$
13.	$Evaluate: \int e^{tan^{-1}x}\left[\frac{(1+x+x^2)}{1+x^2}\right] dx$ $Ans:\ xe^{tan^{-1}x} + c$

14.	Evaluate: $\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{3 + \sin 2x}\right) dx$	Ans: $\frac{1}{4}log3$
15.	Evaluate : $\int_0^{\pi/2} \frac{dx}{1+\sqrt{\tan x}}$	Ans: $\frac{\pi}{4}$
16.	$Evaluate: \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{cosx}{1+e^x} dx$	Ans: 1
17.	Evaluate: $\int_0^{\pi/2} \frac{\cos^2 x \, dx}{1 + 3\sin^2 x}$	Ans: $\frac{\pi}{6}$
18.	If $y = tan^{-1}x, find \frac{d^2y}{dx^2}$ *in terms of* y *alone*	Ans: $-2cos^3 ysiny$
19.	If $x\sqrt{1-y^2} + y\sqrt{1-y^2} = a(x-y), prove \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$	
20.	$Evaluate: \int \frac{\sqrt{x}}{\sqrt{a^3-x^3}} dx$	$Ans: \frac{2}{3} sin^{-1} \frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}} + C$
21.	Evaluate: $\int \frac{sin^6x+cos^6x}{sin^2x\, cos^2x} dx$	Ans : tanx – cotx -3x +c
22.	$Find \int \left(\frac{sinx-xcosx}{x(x+sinx)}\right) dx$	Ans: $\log\frac{x}{x+sinx}$ + c
23.	Using properties of determnants, solve for x if $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$	Ans:x= 1, 3 or -4
24.	Using properties of determnants, prove: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = ab + bc + ca + abc$	
25.	A school wants to award its students for the values of Honesty, Regularity and Hard work with a total cash of Rs. 6000. Three times award money for hard work added to that given for honesty amounts to Rs. 11000. The award money given for honesty and hard work together is double the one given for regularity. Represent the above situation algebraically and find the award money for each value, using matrix method. (Answer: Rs. 500, Rs. 2000, Rs. 3500)	

26.	Evaluate using properties of determinants, prove: $\begin{vmatrix} \sqrt{23}+\sqrt{3} & \sqrt{5} & \sqrt{5} \\ \sqrt{15}+\sqrt{46} & 5 & \sqrt{10} \\ 3+\sqrt{115} & \sqrt{15} & 5 \end{vmatrix}$=0 Answer: 0	
27.	Evaluate using properties of determinants, prove: $\begin{vmatrix} sin^2 23^0 & sin^2 67^0 & cos 180^0 \\ -sin^2 67^0 & -sin^2 23^0 & cos^2 180^0 \\ cos 180^0 & sin^2 23^0 & sin^2 67^0 \end{vmatrix}$=0 Answer: 0	
28.	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{1}{1+4tan^2 x}\, dx$	Answer: $\frac{\pi}{6}$
29.	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{5sinx+3cosx}{sinx+cosx}\, dx$	Answer: 2π
30.	If y= $(logx)^{cosx} + \frac{x^2+1}{x^2-1}, find\, \frac{dy}{dx}$	Answer: $logx^{cosx}\left[\frac{cosx}{xlogx} - sinx \log(logx)\right] - \frac{4x}{(x^2-1)^2}$

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