



INDIAN SCHOOL AL WADI AL KABIR

MATHEMATICS (XII)

Holiday Homework: 2017-18 (Submission date: 7th Aug 2017)

Chapters: Relations and Functions, Inverse Trig, Differentiation and Integration

1 mark

1. Find the value of $\int_{-\pi/2}^{\pi/2} (x^3 + x \cos x + \tan^5 x + 1) dx$
2. Evaluate $\int \tan^2(2x - 3) dx$
3. Determine whether the binary operation $*$ defined below is associative. On Z^+ define $a * b = 2^{ab}$
4. Find the integral of $\int \frac{\sqrt{1+x^2}}{x^4} dx$
5. Evaluate : $\int \cos^{-1}(\sin x) dx$
6. Find $\int_{-1}^1 \sin^5 x \cos^4 x dx$
7. Find $\int_{-1}^1 \sin^5 x \cos^4 x dx$
8. Find $\int_0^{\pi} \sin^7 x dx$

2 mark

9. If $y = \log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$, show that $\frac{dy}{dx} = \operatorname{cosec} x$.
10. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ then find the value of $z + yz + zx$.
11. Evaluate $\int \frac{dx}{x + x \log x}$
12. Evaluate: $\int \frac{(x-4)e^x}{(x-2)^3} dx$

13. Solve : $\tan^{-1} \frac{1-x}{1+x} = \frac{1}{2} \tan^{-1} x, x > 0$

14. Evaluate $\int \frac{1}{1+\cot x} dx$

4 mark

15. Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

16. Show that $f : \mathbb{N} \rightarrow \mathbb{N}$ given by $\begin{cases} n+1, & \text{if } n \text{ is odd} \\ n-1, & \text{if } n \text{ is even} \end{cases}$ is a bijective function.

17. Evaluate :

$$\int \frac{\sin x - x \cos x}{x(x + \sin x)} dx$$

18. Evaluate :

$$\int \frac{x^3}{(x-1)(x^2+1)} dx$$

19. Evaluate :

$$\int_0^{\pi/2} \frac{\cos^2 x dx}{1+3 \sin^2 x}$$

20. Evaluate :

$$\int_0^{\pi/4} \left(\frac{\sin x + \cos x}{3 + \sin 2x} \right) dx$$

21. Evaluate : $\int \frac{3x+1}{\sqrt{5-2x-x^2}} dx$

22. Evaluate : $\int \frac{x^4}{(x-1)(x^2+1)} dx$

23. Evaluate $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

24. Evaluate: $\int_0^{\pi/4} \frac{\cos^2 x}{\cos^2 x + 4 \sin^2 x} dx$

25. If $(a + bx)e^{\frac{y}{x}} = x$ then prove that $x^3 \frac{d^2 y}{dx^2} = \left(x \frac{dy}{dx} - y \right)^2$.

26. A relation R in the set of real numbers is defined as xRy if and only if $x-y + \sqrt{2}$ is an irrational number. Find if R is (i) Reflexive (ii) Symmetric (iii) Transitive.

27. Evaluate: $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

28. If $y = 2 \cos(\log x) + 3 \sin(\log x)$, prove that $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$.

29. Prove that $2 \tan^{-1} \frac{1}{5} + \sec^{-1} \frac{5\sqrt{2}}{7} + 2 \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

30. Evaluate $\int \frac{dx}{x \left[6(\log x)^2 + 7 \log x + 2 \right]}$

6 mark

31. Integrate $\int \frac{x^2 dx}{(x+3)\sqrt{3x+4}}$

32. Evaluate: $\int_{-1}^2 |x^3 - x| dx$

33. Evaluate $\int_0^{\pi} \frac{dx}{a^2 \cos^2 x + b^2 \sin^2 x}$

34. Find the derivative of $y = \log \left(\frac{x + \sqrt{x^2 + a^2}}{x - \sqrt{x^2 - a^2}} \right)$

35. Evaluate : $\int_0^{\frac{3}{2}} |x \cos \pi x| dx$

36. Find : $\int (3x + 1)\sqrt{4 - 3x - 2x^2} dx$

37. Evaluate $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\cos^4 x + \sin^4 x} dx$

38. Evaluate $\int \frac{2x^2 + 1}{x^2(x^2 + 4)} dx$

39. Evaluate : $\int (3x + 2)\sqrt{3x^2 + 4x + 6} dx$

40. Evaluate: $\int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$.