CLASS 11

LEVEL 2

IMO

SOF INTERNATIONAL MATHEMATICS OLYMPIAD 2019-20

DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
3. In the school code column in the OMR Sheet, please fill in code allocated to your school and not the exam center code.
4. The Question Paper comprises two sections : **Mathematics** Section (45 Questions) and **Achievers Section** (5 Questions).

   Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
5. All questions are compulsory. There is no negative marking. Use of calculator / smart phone is not permitted.
6. There is only ONE correct answer. Choose only ONE option for an answer.
7. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.

   Q. 16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is______.

   A. 11.450 kg B. 11.000 kg C. 11.350 kg D. 11.250 kg

   As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet. 16. ● Ⓑ Ⓒ Ⓓ
8. Rough work should be done in the blank space provided in this booklet.
9. Please fill in your personal details in the space provided on this page before attempting the paper.
10. **RETURN THE OMR SHEET AND QUESTION PAPER TO THE INVIGILATOR AT THE END OF THE EXAM.**

SCIENCE OLYMPIAD FOUNDATION

Inspiring Young Minds Through Knowledge Olympiads

Name:..

Section:........... SOF Olympiad Roll No.:.. Contact No.:...

1. If $P = \{a, b, c, d\}$ and $Q = \{1, 2, 3\}$, then which of the following is a relation from P to Q?

A. $R_1 = \{(1, a), (2, b), (3, c)\}$
B. $R_2 = \{(a, 1), (2, b), (c, 3)\}$
C. $R_3 = \{(a, 1), (d, 3), (b, 2), (b, 3)\}$
D. $R_4 = \{(a, 1), (b, 2), (c, 3), (3, d)\}$

2. The value of $(A \cup B \cup C) \cap (A \cap B^c \cap C^c) \cap C^c$ is

A. $B \cap C^c$
B. $A \cap B \cap C$
C. $B \cap C$
D. None of these

3. The general solution of $\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ is

A. $\theta = n\pi + (-1)^{n+1}\frac{\pi}{6}, n \in Z$
B. $\theta = n\pi,\ n \in Z$
C. $\theta = n\pi + (-1)^{n+1}\frac{\pi}{3}, n \in Z$
D. $\theta = \frac{n\pi}{2}, n \in Z$

4. Find locus of point P in such a way that its distances from points A (2, 3) and B (5, –1) are always equal.

A. $4x + 6y = 13$
B. $6x - 8y = 10$
C. $8x - 6y = 10$
D. $6x - 8y = 13$

5. If $a_1, a_2, a_3, ..., a_{4001}$ are terms of an A.P. such that $\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + ... + \frac{1}{a_{4000}a_{4001}} = 10$ and $a_2 + a_{4000} = 50$, then $|a_1 - a_{4001}|$ is equal to

A. 20
B. 30
C. 40
D None of these

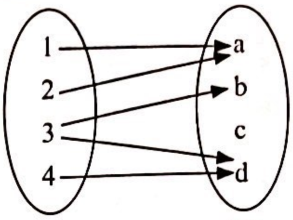
6. If p and q be positive, then the coefficients of x^p and x^q in the expansion of $(1 + x)^{p+q}$ are

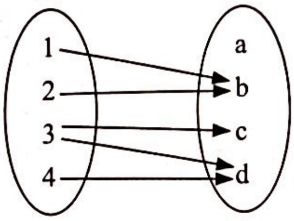
A. Equal
B. Equal in magnitude but opposite in sign
C. Reciprocal to each other
D. None of these

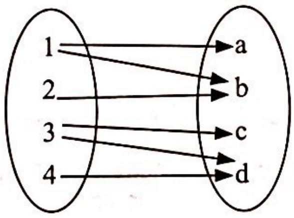
7. For $n \in N$, $x^{n+1} + (x + 1)^{2n-1}$ is divisible by

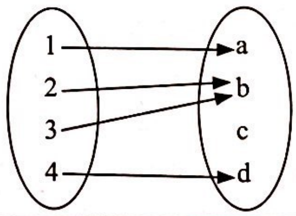
A. x
B. $x + 1$
C. $x^2 - x + 1$
D. $x^2 + x + 1$

8. Which one of the following options represent a function?

A.


B.


C.


D.


9. The solution of the equation $5^{\log_a x} + 5x^{\log_a 5} = 3, a > 0$, is

A. $2^{\log_a 5}$
B. $2^{-\log_5 a}$
C. $2^{-\log_a 5}$
D. None of these

10. If G is the centroid of triangle with vertices $A(a, 0)$, $B(-a, 0)$ and $C(b, c)$, then $\frac{AB^2 + BC^2 + CA^2}{GA^2 + GB^2 + GC^2} =$

A. 1
B. 2
C. 3
D. 4

11. The proposition $\sim p \vee (p \wedge \sim q)$ is equivalent to

A. $p \vee \sim q$
B. $p \rightarrow \sim q$
C. $q \rightarrow p$
D. $p \wedge \sim q$

12. If the roots of the equation $x^2 - px + q = 0$ differ by unity, then

A. $p^2 = 1 - 4q$

B. $p^2 = 1 + 4q$

C. $q^2 = 1 - 4p$

D. $q^2 = 1 + 4p$

13. The sum of 100 observations and the sum of their squares are 400 and 2475, respectively. Later on, three observations, 3, 4 and 5 were found to be incorrect. If the incorrect observations are omitted, then the variance of the remaining observations is

A. 8.00

B. 8.25

C. 9.00

D. 8.50

14. The sum of n terms of the series $\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}+...$, is

A. $\frac{1}{2n+1}$

B. $\frac{2n}{2n+1}$

C. $\frac{n}{2n+1}$

D. $\frac{2n}{n+1}$

15. There are 20 persons among whom two are brothers. The number of ways in which we can arrange them around a circle so that there is exactly one person between the brothers is

A. 19!

B. 2 × 18!

C. 2! × 17!

D. None of these

16. The set of values of θ satisfying the inequation $2\sin^2\theta - 5\sin\theta + 2 > 0$, where $0 < \theta < 2\pi$, is

A. $\left(0,\frac{\pi}{6}\right)\cup\left(\frac{5\pi}{6},2\pi\right)$

B. $\left[0,\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6},2\pi\right]$

C. $\left[0,\frac{\pi}{3}\right]\cup\left[\frac{2\pi}{3},2\pi\right]$

D. None of these

17. If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right)=z\cos\left(\theta+\frac{4\pi}{3}\right)$, then find the value of $xy + yz + zx$.

A. xyz

B. 1

C. 0

D. $\frac{1}{xyz}$

18. If the product of roots of the equation $x^2 - 3kx + 2e^{2\log k} - 1 = 0$ is 7, then its roots will be real when

A. $k = 1$

B. $k = 2$

C. $k = 3$

D. None of these

19. In a college of 300 students, every student reads 5 newspaper and every newspaper is read by 60 students. The number of newspaper is

A. At least 30

B. At most 20

C. Exactly 25

D. None of these

20. If p is the length of the perpendicular from the origin to the line $\frac{x}{a}+\frac{y}{b}=1$, then

A. $p^2 = a^2 + b^2$

B. $\frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}$

C. $p = a + b$

D. $\frac{1}{p}=\frac{1}{a}+\frac{1}{b}$

21. At an election, a voter may vote for any number of candidates, not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is

A. 385

B. 1110

C. 5040

D. 6210

22. The number of integral value(s) of λ for which $x^2 + y^2 + \lambda x + (1 - \lambda)y + 5 = 0$ is the equation of a circle whose radius cannot exceed 5, is

A. 14

B. 18

C. 16

D. None of these

23. If $A(3, 2, -4)$, $B(5, 4, -6)$ and $C(9, 8, -10)$ are three collinear points, then the ratio in which point C divides AB is

A. 3 : 2 externally
B. 3 : 2 internally
C. 2 : 3 externally
D. 2 : 3 internally

24. Solve : $|x - 1| > 2 - 3x$

A. $-1 < x < \frac{-1}{2}$
B. $0 < x < \frac{1}{2}$
C. $-\infty < x < \infty$
D. None of these

25. If the coefficients of p^{th}, $(p+1)^{th}$ and $(p+2)^{th}$ terms in the expansion of $(1+x)^n$ are in A.P., then

A. $n^2 - 2np + 4p^2 = 0$
B. $n^2 - n(4p + 1) + 4p^2 - 2 = 0$
C. $n^2 - n(4p + 1) + 4p^2 = 0$
D. None of these

26. Let P (2, –1, 4) and Q (4, 3, 2) are two points and a point R on PQ is such that $3\,PQ = 5\,QR$, then the coordinates of R are

A. $\left(\frac{14}{5}, \frac{3}{5}, \frac{16}{5}\right)$
B. $\left(\frac{16}{5}, \frac{7}{5}, \frac{14}{5}\right)$
C. $\left(\frac{11}{4}, \frac{1}{2}, \frac{13}{4}\right)$
D. None of these

27. Evaluate : $\lim_{x \to 0} \frac{\sin 2x + \sin 6x}{\sin 5x - \sin 3x}$.

A. 1
B. 2
C. 3
D. 4

28. If a, b, c are p^{th}, q^{th} and r^{th} terms of a G.P., then $(q - r) \log a + (r - p) \log b + (p - q) \log c$ is equal to

A. $p + q + r$
B. 1
C. $-pqr$
D. 0

29. If $z = x + iy$, $z^{1/3} = a - ib$ and $\frac{x}{a} - \frac{y}{b} = \lambda(a^2 - b^2)$, then λ is equal to

A. 1
B. 2
C. 3
D. 4

30. If e and e' are the eccentricities of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$, then the point $\left(\frac{1}{e}, \frac{1}{e'}\right)$ lies on the circle

A. $x^2 + y^2 = 1$
B. $x^2 + y^2 = 2$
C. $x^2 + y^2 = 3$
D. $x^2 + y^2 = 4$

31. How many words can be formed from the letters of the word 'SLAUGHTER' so that all the vowel come together?

A. 30240
B. 720
C. 20240
D. 5040

32. A parabola with vertex (2, 3) and axis parallel to the y-axis passes through (4, 5). Then length of its latus rectum is

A. 5
B. 8
C. 2
D. None of these

33. A person draws a card from a pack of playing cards, replace it and shuffles the pack. He continues doing this until he draws a spade. The chance that he will fail the first two times is

A. $\frac{9}{64}$
B. $\frac{1}{64}$
C. $\frac{1}{16}$
D. $\frac{9}{16}$

34. Common roots of the equations $z^3 + 2z^2 + 2z + 1 = 0$ and $z^{1985} + z^{100} + 1 = 0$ are

A. ω, ω^2
B. ω, ω^3
C. ω^2, ω^3
D. None of these

35. In an ellipse, the distance between its foci is 6 and minor axis is 8. The eccentricity, is

A. $\frac{1}{2}$

B. $\frac{4}{5}$

C. $\frac{1}{\sqrt{5}}$

D. $\frac{3}{5}$

36. If the coefficient of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is

A. 56

B. 55

C. 45

D. 15

37. Consider the following statements :

p : I have the raincoat

q : I can walk in the rain.

The proposition "If I have the raincoat, then I can walk in the rain" is represented by

A. $p \to q$

B. $p \vee q$

C. $p \wedge q$

D. $p \leftrightarrow q$

38. Seven white balls and three black balls are randomly placed in a row. The probability that no two black balls are placed adjacently, is

A. $\frac{1}{2}$

B. $\frac{7}{15}$

C. $\frac{2}{15}$

D. $\frac{1}{3}$

39. Read the statements carefully and select the correct option.

Statement-I : $(A \cup B) - (A \cap B)$ is equal to $(A - B) \cup (B - A)$.

Statement-II : $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

A. Both Statement-I and Statement-II are true.

B. Statement-I is true but Statement-II is false.

C. Statement-I is false but Statement-II is true.

D. Both Statement-I and Statement-II are false.

40. If the mean of 10 observations is 50 and the sum of the squares of the deviations of the observations from the mean is 250, then the coefficient of variation of those observations is

A. 25

B. 50

C. 10

D. 5

41. Which of the following results is valid ?

A. $(1 + x)^n > (1 + nx)$ for all natural numbers n

B. $(1 + x)^n \geq (1 + nx)$ for all natural numbers n, where $x > -1$

C. $(1 + x)^n \leq (1 + nx)$ for all natural numbers n

D. $(1 + x)^n < (1 + nx)$ for all natural numbers n

42. If $y = \frac{1}{1+x+x^2}$, then $\frac{dy}{dx}$ is equal to

A. $y^2(1 + 2x)$

B. $\frac{-(1+2x)}{y^2}$

C. $\frac{(1+2x)}{y^2}$

D. $-y^2(1 + 2x)$

43. Centre of the circle whose radius is 3 and which touches internally the circle $x^2 + y^2 - 4x - 6y - 12 = 0$ at the point $(-1, -1)$, is

A. $\left(\frac{7}{5}, \frac{-4}{5}\right)$

B. $\left(\frac{4}{5}, \frac{7}{5}\right)$

C. $\left(\frac{3}{5}, \frac{4}{5}\right)$

D. $\left(\frac{7}{5}, \frac{3}{5}\right)$

44. The domain of the function f defined by $f(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$ is equal to

A. $(-\infty, -1) \cup (1, 4]$

B. $(-\infty, -1] \cup (1, 4]$

C. $(-\infty, -1) \cup [1, 4]$

D. $(-\infty, -1) \cup [1, 4)$

45. 10 mangoes are to be distributed among 5 persons. The probability that at least one of them will receive none, is

A. $\frac{35}{143}$

B. $\frac{108}{143}$

C. $\frac{18}{143}$

D. $\frac{125}{143}$

ACHIEVERS SECTION

46. Study the following statements carefully and select the correct option.

Statement-I : Consider $|z_1| = 1$, $|z_2| = 2$ and $|z_3| = 3$. If $|z_1 + 2z_2 + 3z_3| = 6$, then the value of $|z_2z_3 + 8z_3z_1 + 27z_1z_2|$ is 36.

Statement-II : The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is $\frac{1}{5+3\cos\theta}$.

A. Both Statement-I and Statement-II are true.

B. Statement-I is true but Statement-II is false.

C. Statement-I is false but Statement-II is true.

D. Both Statement-I and Statement-II are false.

47. Match the following and select the correct option.

	Column-I		Column-II
(P)	If $\cos(A-B) = 3/5$ and $\tan A \tan B = 2$, then the value of $\cos A \cos B$ is	(1)	$\frac{-1}{3}$
(Q)	If $\theta = \pi/4n$ (n is a positive integer), then the value of $\tan\theta\tan2\theta\tan3\theta\ldots\tan2(n-1)\theta\tan(2n-1)\theta$ is	(2)	1
(R)	If $\cos\alpha = 2\cos\beta$, then the value of $\tan\left(\frac{\alpha-\beta}{2}\right)\tan\left(\frac{\alpha+\beta}{2}\right)$	(3)	$\frac{1}{5}$

	(P)	(Q)	(R)
A.	(2)	(1)	(3)
B.	(3)	(2)	(1)
C.	(2)	(3)	(1)
D.	(3)	(1)	(2)

48. In how many ways can a committee of eight be selected from a group of 10 men and 12 women, such that in the committee,

(a) there are three men and five women.

(b) these are at least six women.

	(a)	(b)
A.	55440	15345
B.	95040	15345
C.	55440	49995
D.	95040	49995

49. If A, B and C are three events, then which of the following is incorrect?

A. P (Exactly two of A, B, C occur) $\leq P(A \cap B) + P(B \cap C) + P(C \cap A)$

B. $P(A \cup B \cup C) \leq P(A) + P(B) + P(C)$

C. P (A and at least one of BC, occurs) $\geq P(A \cap B) + P(A \cap C)$

D. None of these

50. Read the following statements carefully and state T for true and F for false.

(P) The coefficient of $\frac{1}{x^{17}}$ in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is – 1365.

(Q) If n is even, then the middle term in the expansion of $\left(x^2 + \frac{1}{x}\right)^n$ is $924x^6$, then n is equal to 12.

(R) The term independent of x in the expansion of $\left(3x - \frac{2}{x^2}\right)^{15}$ is $3003\ (3)^5(2)^{10}$.

	(P)	(Q)	(R)
A.	T	T	T
B.	F	F	T
C.	T	T	F
D.	F	F	F

SPACE FOR ROUGH WORK