CLASS 12

LEVEL 2

IMO

SOF INTERNATIONAL MATHEMATICS OLYMPIAD 2019-20

DO NOT OPEN THIS BOOKLET UNTIL ASKED TO DO SO

Total Questions: 50 | Time: 1 hr.

Guidelines for the Candidate

1. You will get additional ten minutes to fill up information about yourself on the OMR Sheet, before the start of the exam.
2. Write your **Name, School Code, Class, Section, Roll No.** and **Mobile Number** clearly on the **OMR Sheet** and do not forget to sign it. We will share your marks / result and other information related to SOF exams on your mobile number.
3. In the school code column in the OMR Sheet, please fill in code allocated to your school and not the exam center code.
4. The Question Paper comprises two sections : **Mathematics** Section (45 Questions) and **Achievers Section** (5 Questions).

   Each question in Achievers Section carries 3 marks, whereas all other questions carry one mark each.
5. All questions are compulsory. There is no negative marking. Use of calculator / smart phone is not permitted.
6. There is only ONE correct answer. Choose only ONE option for an answer.
7. To mark your choice of answers by darkening the circles on the OMR Sheet, use **HB Pencil** or **Blue / Black ball point pen** only. E.g.

   Q. 16: Rahul bought 4 kg 90 g of apples, 2 kg 60 g of grapes and 5 kg 300 g of mangoes. The total weight of all the fruits he bought is______.

   A. 11.450 kg B. 11.000 kg C. 11.350 kg D. 11.250 kg

   As the correct answer is option A, you must darken the circle corresponding to option A on the OMR Sheet. 16. ● Ⓑ Ⓒ Ⓓ
8. Rough work should be done in the blank space provided in this booklet.
9. Please fill in your personal details in the space provided on this page before attempting the paper.
10. **RETURN THE OMR SHEET AND QUESTION PAPER TO THE INVIGILATOR AT THE END OF THE EXAM.**

SOF

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1. If $f(x) = \frac{2^x - 2^{-x}}{2^x + 2^{-x}}$, then $f^{-1}(x)$ is

A. $\frac{1}{2}\log_2 \frac{x}{1-x}$

B. $\frac{1}{2}\log_2 \frac{1+x}{1-x}$

C. $\frac{1}{2}\log_2 \frac{1+x}{x}$

D. $\frac{1}{2}\log_2 \frac{2+x}{2-x}$

2. If $f(x)$ is defined on $[-2, 2]$ by $f(x) = 4x^2 - 3x + 1$ and $g(x) = \frac{f(-x) - f(x)}{(x^2+3)}$, then $\int_{-2}^{2} g(x)dx =$

A. 64

B. – 48

C. 0

D. 24

3. If Rolle's theorem hold for the function $f(x) = x^3 + bx^2 + cx$, $1 \leq x \leq 2$ at the point 4/3, then find the values of b and c respectively.

A. 8, – 5

B. –5, 8

C. 5, –8

D. –5, –8

4. The derivative of $\operatorname{cosec}^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at $x = \frac{1}{2}$, is

A. – 4

B. 4

C. – 1

D. None of these

5. If $x > 0$, $y > 0$ and $x > y$, then $\tan^{-1}(x/y) + \tan^{-1}[(x+y)/(x-y)]$ is equal to

A. $\pi/2$

B. $\pi/4$

C. $3\pi/4$

D. None of these

6. For positive numbers x, y and z, the numerical value of the determinant $\begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$ is ______.

A. 1

B. 0

C. – 1

D. log xyz

7. $\int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$ is equal to

A. $\frac{2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + C$

B. $\frac{2}{\sqrt{\tan x}} + \frac{3}{2}(\tan x)^{3/2} + C$

C. $\frac{1}{\sqrt{\tan x}} + \frac{3}{2}(\tan x)^{2/3} + C$

D. $-\frac{2}{\sqrt{\tan x}} + \frac{2}{3}(\tan x)^{3/2} + C$

8. Let W denote the words in the English dictionary. Define the relation R by $R = \{(x, y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$. Then R is

A. Symmetric and transitive only.

B. Reflexive and symmetric only.

C. An equivalence relation.

D. Reflexive and transitive only.

9. The area bounded by the curve $y = 6 + 4x - x^2$ and the line $2x - y = 2$ is

A. 12 sq. units

B. 24 sq. units

C. 36 sq. units

D. 48 sq. units

10. Equation of line passing through $A(1, 0, 3)$, intersecting the line $\frac{x}{2} = \frac{y-1}{3} = \frac{z-2}{1}$ and parallel to the plane $x + y + z = 2$ is

A. $\frac{3x-1}{2} = \frac{2y-3}{3} = \frac{2z-5}{-1}$

B. $\frac{x-1}{2} = \frac{y-0}{3} = \frac{z-3}{-1}$

C. $\frac{x-(2/3)}{1} = \frac{y-(3/2)}{0} = \frac{z+(1/2)}{3}$

D. $\frac{3x-1}{2} = \frac{2y-3}{-3} = \frac{6z-13}{5}$

11. For two events A and B, if $P(A) = P(A/B) = \frac{1}{4}$ and $P(B/A) = \frac{1}{2}$, then which of the following is not true?

A. A and B are independent

B. $P(A'/B) = \frac{3}{4}$

C. $P(B'/A') = \frac{1}{2}$

D. None of these

12. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$ is

A. $\frac{3}{17}$

B. $\frac{4}{17}$

C. $\frac{5}{17}$

D. $\frac{6}{17}$

13. The radius of the base of a cone is increasing at the rate of 3 cm/min and the altitude is decreasing at the rate by 4 cm/min. The rate of change of lateral surface area when the radius is 7 cm and altitude is 24 cm, is

A. 54π cm^2 / min

B. 7π cm^2 / min

C. 27π cm^2 / min

D. None of these

14. Given $f(x) = \log\frac{1+x}{1-x}$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $fog(x)$ equals

A. $-f(x)$

B. $3\,f(x)$

C. $[f(x)]^3$

D. $[f(x)]^2$

15. A line makes acute angles of α, β and γ with the coordinate axes such that $\cos\alpha\cos\beta = \cos\beta\cos\gamma = \frac{2}{9}$ and $\cos\gamma\cos\alpha = \frac{4}{9}$, then $\cos\alpha + \cos\beta + \cos\gamma$ is equal to

A. $\frac{25}{9}$ B. $\frac{5}{9}$

C. $\frac{5}{3}$ D. $\frac{2}{3}$

16. The degree of the differential equation satisfying the relation $\sqrt{1+x^2} + \sqrt{1+y^2} = \lambda\left(x\sqrt{1+y^2} - y\sqrt{1+x^2}\right)$ is

A. 1

B. 2

C. 3

D. None of these

17. Let $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 4\hat{k}$. A vector $\vec{r}$ satisfying $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$ is

A. $-2\hat{i} + 2\hat{j} + 2\hat{k}$

B. $-2\hat{i} + \hat{j} + 3\hat{k}$

C. $-2\hat{i} - \hat{j} + 5\hat{k}$

D. $\hat{i} - 5\hat{j} + 3\hat{k}$

18. $\int \frac{1}{x\{6(\log x)^2 + 7\log x + 2\}}dx$ is equal to

A. $\log\left|\frac{2\log x - 1}{3\log x + 2}\right| + C$

B. $\log\left|\frac{3\log x + 1}{2\log x + 2}\right| + C$

C. $\log\left|\frac{\log x + 1}{\log x + 2}\right| + C$

D. None of these

19. Domain of the definition of the function $f(x) = \sqrt{\sin^{-1}(2x) + \frac{\pi}{6}}$ is

A. $\left[\frac{-1}{4}, \frac{1}{2}\right]$

B. $\left[\frac{-1}{2}, \frac{1}{9}\right]$

C. $\left[\frac{-1}{2}, \frac{1}{2}\right]$

D. $\left[\frac{-1}{4}, \frac{1}{4}\right]$

20. The curves $x^3 - 3xy^2 = a$ and $3x^2y - y^3 = b$, where a and b are constants, cut each other at an angle of

A. $\frac{\pi}{3}$

B. $\frac{\pi}{4}$

C. $\frac{\pi}{2}$

D. None of these

21. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors and λ is a real number, then the vectors $\vec{a} + 2\vec{b} + 3\vec{c}, \lambda\vec{b} + 4\vec{c}$ and $(2\lambda - 1)\vec{c}$ are non-coplanar for

A. No value of λ

B. All except one value of λ

C. All except two values of λ

D. All values of λ.

22. The distance (in units) of the origin from the plane through the points (1, 1, 0), (1, 2, 1) and (–2, 2, –1) is

A. $\frac{3}{\sqrt{11}}$

B. $\frac{5}{\sqrt{22}}$

C. 3

D. $\frac{4}{\sqrt{22}}$

23. If $(x^2 - 1)\frac{dy}{dx} + 2xy = x$, $y(0) = 0$, then $y(2) =$

A. 1 B. $\frac{1}{3}$

C. $\frac{2}{3}$ D. 2

24. The area bounded by the straight lines $x = 0$, $x = 2$ and the curves $y = 2^x$, $y = 2x - x^2$ is

A. $\left(\frac{4}{3} - \frac{1}{\log 2}\right)$ sq. units

B. $\left(\frac{3}{\log 2} + \frac{4}{3}\right)$ sq. units

C. $\left(\frac{4}{\log 2} - 1\right)$ sq. units

D. $\left(\frac{3}{\log 2} - \frac{4}{3}\right)$ sq. units

25. If from each of 3 boxes containing 3 white and 1 black, 2 white and 2 black, 1 white and 3 black balls, one ball is drawn, then the probability of drawing 2 white and 1 black ball is

A. $\frac{13}{32}$

B. $\frac{1}{4}$

C. $\frac{1}{32}$

D. $\frac{3}{16}$

26. Function $f(x) = \begin{cases} |x+1|, & x < -2 \\ 2x+3, & -2 \le x < 0 \\ x^2+3 & 0 \le x < 3 \\ x^3 - 15, & x \ge 3 \end{cases}$ is

A. Continuous at all points in R.

B. Continuous only at $x = -2$.

C. Continuous at all points in R except at $x = -2$.

D. None of these

27. The minimum value of $z = 3x + 4y$ subject to $3x + 5y \ge 30$, $x + y \ge 8$, $x, y \ge 0$ is

A. 25

B. 27

C. 30

D. 32.

28. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$, then

A. A is zero matrix

B. $A^2 = I$

C. A^{-1} does not exist

D. $A = (-1) I$

29. The area of the portion of the circle $x^2 + y^2 = 64$ which is exterior to the curve $y^2 = 12x$ is

A. $\frac{8}{3}(8\pi + \sqrt{3})$ sq. units

B. $\frac{8}{3}(8\pi - \sqrt{3})$ sq. units

C. $\frac{16}{3}(8\pi + \sqrt{3})$ sq. units

D. None of these

30. If a, b, c are non-zeros, then the system of equations

$(\alpha + a)x + \alpha y + \alpha z = 0$

$\alpha x + (\alpha + b)y + \alpha z = 0$

$\alpha x + \alpha y + (\alpha + c)z = 0$

has a non-trivial solution, if

A. $2\alpha = a + b + c$

B. $\alpha^{-1} = a + b + c$

C. $\alpha + a + b + c = 1$

D. $\alpha^{-1} = -(a^{-1} + b^{-1} + c^{-1})$

31. If m and M are the least and greatest values of $(\cos^{-1}x)^2 - (\sin^{-1}x)^2$, then $\frac{M}{m} =$

A. 1

B. – 1

C. 3

D. – 3

32. The foot of the perpendicular from the point (1, 2, 3) on the line $\vec{r} = (6\hat{i} + 7\hat{j} + 7\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$ has the coordinates

A. (5, 8, 15)

B. (8, 5, 15)

C. (3, 5, 9)

D. (3, 5, –9)

33. The value of $\int \frac{x^7}{(1-x^2)^5} dx$, is

A. $\frac{x^8}{(1-x^2)^4} + C$

B. $\frac{1}{8}\frac{x^8}{(1-x^2)^4} + C$

C. $\frac{1}{8}\frac{x^4}{(1-x^2)^4} + C$

D. None of these

34. Let $f(x) = [3 + 2\cos x], x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, where $[\cdot]$ denotes the greatest integer function. The number of points of discontinuity of $f(x)$ is

A. 3
B. 2
C. 5
D. None of these

35. The number of positive solutions satisfying the equation $\tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right)$, is

A. 1
B. 2
C. 8
D. 9

36. If $0 < P(A) < 1$, $0 < P(B) < 1$ and $P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$, then select the correct option

A. $P(B/A) = P(B) - P(A)$
B. $P(A/B) = P(A) - P(B)$
C. $P(\overline{A \cup B}) = P(\bar{A}) \cdot P(\bar{B})$
D. None of these

37. If a tangent to the curve $y = 6x - x^2$ is parallel to the line $4x - 2y - 1 = 0$, then the point of tangency on the curve is

A. (2, 8)
B. (8, 2)
C. (6, 1)
D. (4, 2)

38. If $y = \sin(\log_e x)$, then $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} =$

A. $\sin(\log_e x)$
B. $\cos(\log_e x)$
C. y^2
D. $-y$

39. If $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar vectors such that $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$, $\vec{c} \times \vec{a} = \vec{b}$, then $|\vec{a}| + 2|\vec{b}| - 3|\vec{c}|$ is equal to

A. 1
B. 0
C. 2
D. None of these

40. A company manufactures two types of chemicals A and B. Each chemical requires two types of raw material P and Q. The table below shows number of units of P and Q required to manufacture one unit of A and one unit of B and the total availability of P and Q.

Raw material \ Chemical	A	B	Availability
P	3	2	120
Q	2	5	160

The company gets profit of ₹ 350 and ₹ 400 by selling one unit of A and one unit of B respectively. How many units of the chemicals A and B respectively should be manufactured so that the company gets maximum profit? (Assume that the entire production of A and B can be sold.)

A. $\frac{280}{11}, \frac{240}{11}$

B. $\frac{280}{13}, \frac{210}{13}$

C. $\frac{240}{13}, \frac{210}{13}$

D. $\frac{210}{11}, \frac{240}{11}$

41. If $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2}\right] dx = a + \frac{b}{\log 2}$, then

A. $a = e, b = -2$
B. $a = e, b = 2$
C. $a = -e, b = 2$
D. None of these

42. If M is the skew symmetric matrix of order $n \times n$, then select the correct option.

A. $\det(M - I) = \det(M + I)$
B. $\det(M - I) = -\det(M + I)$, if n is odd
C. $\det(M - I) = \det(M + I)$, if n is even
D. Both B and C

43. The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of a ΔABC. The length of the median through A is

A. $\frac{\sqrt{34}}{2}$ units

B. $\frac{\sqrt{48}}{2}$ units

C. $\sqrt{18}$ units

D. None of these

44. The general solution of the differential equation $\left[2\sqrt{xy}-x\right]dy+y\,dx=0$ is

A. $\log x+\sqrt{\frac{y}{x}}=c$

B. $\log y-\sqrt{\frac{x}{y}}=c$

C. $\log y+2\sqrt{\frac{x}{y}}=c$

D. None of these

45. The value of determinant $\begin{vmatrix}\cos\alpha & -\sin\alpha & 1\\ \sin\alpha & \cos\alpha & 1\\ \cos(\alpha+\beta) & -\sin(\alpha+\beta) & 1\end{vmatrix}$ is

A. Independent of α

B. Independent of β

C. Independent of α and β

D. None of these

ACHIEVERS SECTION

46. Read the following statements carefully and select the correct option. Let $f(x)=\tan^{-1}x$ and $g(x)=x-\frac{x^3}{6}$.

Statement-1 : $f(x)<g(x)$ for $0<x\leq 1$

Statement-2 : $h(x)=\tan^{-1}x-x+\frac{x^3}{6}$ decreases on $[-1, 1]$.

A. Statement-1 is true but Statement-2 is false.

B. Both Statement-1 and Statement-2 are true.

C. Both Statement-1 and Statement-2 are false.

D. Statement-1 is false but Statement-2 is true.

47. Which of the following options is correct?

A. The vector equation of the plane passing through the origin and the line of intersection of the plane $\vec{r}\cdot\vec{a}=\lambda$ and $\vec{r}\cdot\vec{b}=\mu$ is $\vec{r}\cdot(\lambda\vec{b}+\mu\vec{a})=0$.

B. Shortest distance between the lines $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{1}$ and $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{1}$ is equal to $\sqrt{2}$.

C. The point on the line $\frac{x-2}{1}=\frac{y+3}{-2}=\frac{z+5}{-2}$ at a distance of 6 units from the point (2, –3, –5) is (4, 7, –9).

D. None of these

48. Match the following and select the correct option.

	Column-I		Column-II
(P)	The probability that Krishna will be alive 10 years hence is $\frac{7}{15}$ and that Hari will be alive is $\frac{7}{10}$. The probability that both Krishna and Hari will be dead 10 years hence is	(i)	3/5
(Q)	A bag contains 4 balls. Two balls are drawn at random and are found to be white. The probability that all balls are white is	(ii)	11/50
(R)	A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both hearts. The probability of the missing card to be a heart is	(iii)	4/25

A. (P) → (iii), (Q) → (ii), (R) → (i)

B. (P) → (ii), (Q) → (i), (R) → (iii)

C. (P) → (iii), (Q) → (i), (R) → (ii)

D. (P) → (ii), (Q) → (iii), (R) → (i)

49. Find the value of x, if

(i) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)=4$

(ii) $3\tan^{-1}x+\cot^{-1}x=\pi$

	(I)	(II)
A.	tan 8	–1
B.	tan 8	1
C.	tan 4	1
D.	tan 4	–1

50. Read the following statements carefully and state T for true and F for false.

P. If A and B are two symmetric matrices of order 3, then $A(BA)$ and $(AB)A$ are also symmetric.

Q. The equation $\begin{vmatrix} x & -2 & 1 \\ 2 & x & -3 \\ -1 & 3 & x \end{vmatrix}=0$ has only one real root.

R. If $A=\begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, then adj $A = A$.

	P	Q	R
A.	T	T	F
B.	F	T	T
C.	F	F	T
D.	T	T	T

SPACE FOR ROUGH WORK