INDIAN SCHOOL AL WADI AL KABIR

Sample Paper-2 (2020-2021)

Class: XII | Sub: **MATHEMATICS (041)** | Max Marks: 80
Date: 20.12.2020 | Time: 3 hours

General Instructions:

1. This question paper contains two parts A and B. Each part is compulsory.
2. Part A carries 24 marks and Part B carries 56 marks.
3. Part-A has Objective Type Questions and Part -B has Descriptive Type Questions,
4. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections- I and II.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

1. It consists of three sections- III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section –III, 2 questions of Section IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

Section A

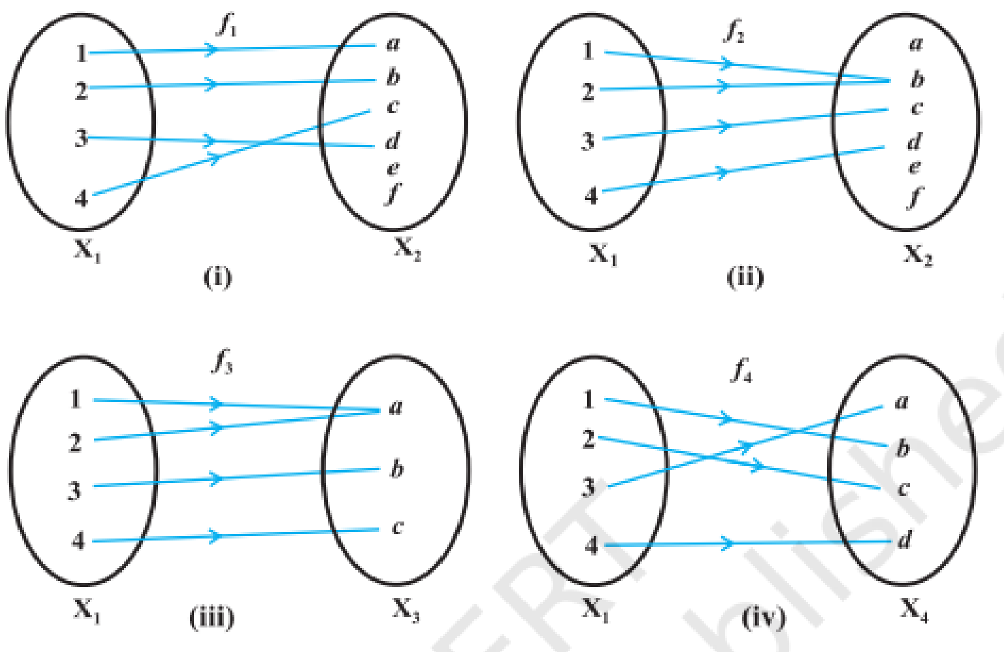
Section- I (1 mark each)

Q1. Find the number of reflexive relations from set A = $\{1, 2, 3\}$ to itself.

OR

Show that the greatest integer function $f: R\ to\ R\ defined\ by\ f(x) = [x]$ is neither one to one nor onto.

Q2. Which of the following function(s) is invertible?



Q3. Write all the equivalence relations in roster form in the set $\{1, 2, 3\}$ $containing$ $(1,2) and$ $(2, 1)$

Q4. Write $A^{-1} if\ A = \begin{bmatrix} 4 & 2 \\ 5 & 5 \end{bmatrix}$.

Q5. Write a $3 \times 3\ matrix\ such\ that\ A = [a_{ij}], such\ that\ a_{ij} = \frac{(i+j)^2}{2}$.

Q6. Using matrices, find k if (1, 4), (k, 0) and (3, 8) are collinear.

Q7. Evaluate: $\int cos^{-1}(sinx)dx$

OR

Evaluate: $\int_0^{2\pi} cos^3x\, dx$

Q8. Find the area of region bounded by $y = x, x = -2,\ x = 2\ and\ the\ X\ axis.$

Q9. Find the general solution of the differential equation $dy + dx = 0.$

OR

Write integrating factor of the differential equation : $x\frac{dy}{dx} + \frac{y}{logx} = \text{xsinx}$

Q10. Find the slope of the normal to the curve $x = 1 - asin\theta, y = acos^2\theta\ at\ \theta = \frac{\pi}{2}.$

Q11. $Find\ a\ vector\ \ of\ magnitude\ 5\ in\ the\ direction\ of\ 3\hat{\imath} - 4\hat{\jmath} + 5\sqrt{3}\hat{k}$

Q12. If a line makes an angle of 30^0, 60^0, 90^0 with the positive direction of x, y, x axes respectively, then find its direction cosines.

Q13 Find the value of λ if $2\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}\ and\ \hat{\imath} + 2\hat{\jmath} - 3\hat{k}\ are\ orthogonal.$

Q14. Find the value(s) of x if $\begin{vmatrix} 2x & 9 \\ 2 & x \end{vmatrix} = \begin{vmatrix} x & -1 \\ 7 & x \end{vmatrix}$

Q15. Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are red.

Q16. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

Section- II

Both the Case study-based questions are compulsory. Attempt any 4 sub parts from each question (17(i to v) and 18 (i to v)-26). Each question carries 1 mark

Q17. On his birthday Hari decided to donate some money to children of an orphanage home. If there were 10 children less, everyone would have got ₹ 30 more. However, if there were 10 children more, everyone would have got ₹ 20 less.

Based on the above information answer the following:

(i) The algebraic equations in terms of x and y are

A	x+ y =30; x-y =20	**B**	3x-y=30; 2x-y= -20	**C**	x-3y =30; x-2y = -20	**D**	x+3y = 300; 2x+y = 20

(ii) Which of the following represents the matrix form of the algebraic equations?

A $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}30\\ -20\end{pmatrix}$ **B** $\begin{pmatrix}3 & -1\\ 2 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}30\\ -20\end{pmatrix}$ **C** $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}30\\ -20\end{pmatrix}$ **D** $\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}30\\ -20\end{pmatrix}$

(iii) The number of students in the orphanage is

A	20	**B**	30	**C**	40	**D**	50

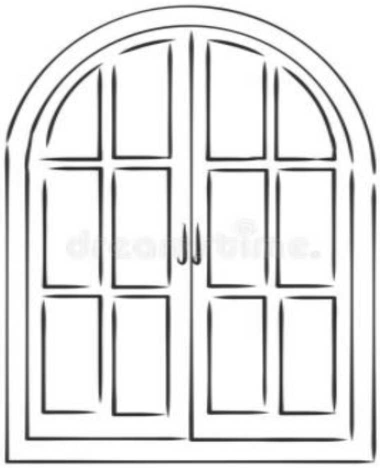
(iv) Amount received by each child is ₹__________

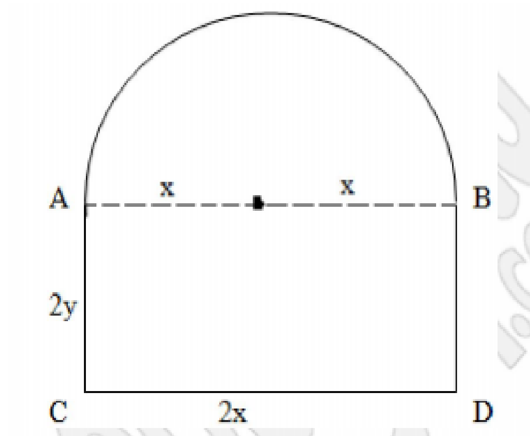
A	90	**B**	100	**C**	120	**D**	150

(v) Total amount donated I ₹ ____________

A	6000	**B**	5000	**C**	7500	**D**	10000

Q18. One carpenter designs a window in the shape of a rectangle which is surmounted by a semicircular opening. The perimeter of the window is 10m





Based on the above information answer the following.

2x and 2y represents the length and breadth of the rectangle.

(i) Which of the following is correct?

A $4x + 2y + \pi x = 10$ **B** $2x + 4y + 2\pi x = 10$ **C** $2x + 4y + \pi x = 10$ **D** $4x + 4y + 2\pi x = 10$

(ii) $Combined\ aea\ of\ rectangle\ and\ semicircle\ is$ _______ Sq.m.

A $4xy + \frac{1}{2}\pi x^2$ **B** $2xy + \frac{1}{2}\pi x^2$ **C** $4xy + \pi x^2$ **D** $4xy + 2\pi x^2$

(iii) For maximizing the area of the window so that maximum light input is possible when length of the rectangle is __________m.

A $\frac{20}{4+\pi}$ **B** $\frac{10}{4+\pi}$ **C** $\frac{4}{10+\pi}$ **D** $\frac{4}{20+\pi}$

(iv) What is the maximum area of the semictcular portion in sq.m?

A. $\frac{100\pi}{(4+\pi)^2}$ **B.** $\frac{50\pi}{4+\pi}$ **C.** $\frac{50\pi}{(4+\pi)^2}$ **D.** $\frac{4\pi}{(10+\pi)^2}$

(v) What is the maximum area of the whole window in sq. m??

A. $\frac{50}{\pi - 4}$ **B.** $\frac{50}{4+\pi}$ **C.** $\frac{100}{4+\pi}$ **D.** $\frac{50}{4-\pi}$

Part- B

Section -III(2marks each)

Q19. Evaluate: $\tan\left(\frac{1}{2}cos^{-1}\frac{2}{\sqrt{5}}\right)$).

Q20. $Find\ the\ matrix\ X\ so\ that\ X\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix} = \begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$.

OR

Find x , y and z if $A = \begin{bmatrix}0 & 2y & z\\ x & y & -z\\ x & -y & z\end{bmatrix} satisfies\ A^T = A.$

Q21. Find the value k if $f(x) = \begin{cases}\frac{cos2x-1}{cosx-1}.x \neq 0\\ k,\ x = 0\end{cases}$ is continous at x= 0.

Q22. $Evaluate: \int e^{tan^{-1}x}\left(\frac{1+x+x^2}{1+x^2}\right)dx$ OR Evaluate: $\int_2^8 \frac{\sqrt{10-x}}{\sqrt{x}+\sqrt{10-x}}dx$

Q23. Find the area of the region bounded by $x = \sqrt{y}\,, the\ lines\ y = 1\,, y = 3\ and\ the\ y\ axis.$

Q24. Solve the differential equation: $cos^2x\frac{dy}{dx} + y = 1$

Q25. Find the difference between the greatest and least value of the function $f(x) = sin2x - x\,, x\epsilon\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

Q26. Find all vectors of magnitude $10\sqrt{3}$ that are perpendicular to the plane of $\hat{\imath} + 2\hat{\jmath} + \hat{k}\ and$ $-\hat{\imath} + 3\hat{\jmath} + 4\hat{k}.$

Q27. Find the equation of the plane passing through the points (2, 1, 0), (3, -3, -3) and (3, 1, 7).

Q28. 10% of the bulbs produced in a factory are of red colour and 2% are red and defective. If one bulb is picked up at random, determine the probability that it is defective given that it is red.

OR

Two cards are selected at random from a well shuffled pack of 52 cards. Find the probability distribution of number of aces.

Section IV

ANSWER THE FOLLOWING (3 marks each)

Q29. Consider $f: [0. \infty) \to [-5. \infty)$ $is\ given\ by\ f(x) = 9x^2 + 6x - 5.$
$Show\ that\ f\ is\ one\ to\ one\ and\ onto.$

Q30. Find $\frac{dy}{dx}, if\ y = (x)^{sinx} + (sinx)^x.$

Q31. If $y = sin^{-1}x$, then show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$

OR

If $x = a(cost + tsint), y = a(sint - tcost)$ $prove$ $\frac{d^2y}{dx^2} = \frac{sec^3t}{at}$

Q32. Find the intervals in which the function $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$
$is\ (i)\ strictly\ decreasing\ (ii)\ strictly\ increasing.$

Q33. $Evaluate: \int \log(logx) + \frac{1}{(logx)^2} dx$

Q34. Find the area enclosed by the curve $y = \sqrt{9 - x^2}$, x= -1 and x= 1

OR

$Evaluate: \int_{-1}^{\frac{3}{2}} |xsin\pi x|\, dx$

Q35. Solve the differential equation: $\frac{dy}{dx} = \frac{ycos(\frac{y}{x}) + x}{xcos(\frac{y}{x})}.$

Section V (5marks each)

Q36. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1 - t)\hat{\imath} + (t - 2)\hat{\jmath} + (3 - t)\hat{k}$

and $\vec{r} = (s + 1)\hat{\imath} + (2s - 1)\hat{\jmath} - (2s + 1)\hat{k}$.

OR

Find equation of a plane through the intersection of planes 3x- y+2z - 4=0 and x+ y+ z -2 = 0 and the point (2, 2, 1).

Q37. If a machine is correctly set up, it produces 90% acceptable items. If it is incorrectly set up, it produces only 40% acceptable items. Past experience shows that 80% of the set ups are correctly done. If after a certain set up, the machine produces 2 acceptable items, find the probability that the machine is correctly setup

OR

A car manufacturing factory has two plants, X and Y. Plant X manufactures 70% of cars and plant Y manufactures 30%. 80% of cars at plant X and 90% of the cars at plant Y are rated of standard quality. A car is chosen at random and is found to be of standard quality. What is the probability that it has come from plant X?

Q38. Solve the following LPP graphically:

$Minimise: Z = 200x + 500y$

Subject to the constrains: $x + 2y \geq 10, x \geq 0, y \geq 0 ,3x + 4y \leq 24.$

OR

A merchant plans to sell two types of personal computers, a desktop model and a portable model that will cost ₹25000 and ₹ 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get the maximum profit, if he does not want to invest more than ₹ 70 lakh and his profit on the desktop model is ₹ 4500 and on the portable model is ₹ 5000. Make an LPP and solve it graphically.

Answer									
Q1.	64	**Q2.**	f_4	**Q3.**	{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}, $\{(1,1),(1,2),(2,3)(2,1),(2,2),(2{,}3)(3,1),(3,2),(3,3)\}$				
Q4.	$\frac{1}{2}\begin{bmatrix}5 & -2\\-5 & 4\end{bmatrix}$	**Q5.**	$\begin{pmatrix}2 & \frac{9}{2} & 8\\ \frac{9}{2} & 8 & \frac{25}{2}\\ 8 & \frac{25}{2} & 18\end{pmatrix}$	**Q6.**	K = -1	**Q9.**	***I*** = $\frac{\pi}{2}x-\frac{x^2}{2}+C$ OR ***I*** = 0		
Q10.	-2	**Q11.**	$\frac{1}{2}(3\hat{\imath}-4\hat{\jmath}+5\sqrt{3}\hat{k})$	**Q12**	$\pm<\frac{\sqrt{3}}{2},\frac{1}{2},0>$	**Q.13.**	$\frac{13}{2}$		
Q14.	±5	**Q15.**	$\frac{25}{102}$	**Q16.**	$\frac{\pi}{2}$				
Q17(i)	B	**Q17(ii)**	B	**Q17(iii)**	**D**	**Q17(iv)**	C	**Q17(v)**	A
Q18(i)	C	**Q18(ii)**	A	**Q18(iii)**	**A**	**Q18(iv)**	B	**Q18(v)**	A
Q19.	$\sqrt{5}-2$	Q20.	X= $\begin{pmatrix}1 & -2\\2 & 0\end{pmatrix}$	$\boldsymbol{x=y=z=0}$	**Q21.**	K= 4			
Q22.	$I=xe^{tan^{-1}x}+c$	$I=3$	**Q23.**	$\frac{2}{3}(3\sqrt{3}-1)$					
Q24.	$ye^{tanx}=e^{tanx}+c$	**Q25.**	π	**Q26.**	$\pm10(\hat{\imath}-\hat{\jmath}+\hat{k})$				
Q27.	21x+9y-3z-51=0	**Q28.**	$\frac{1}{5}$	X	0	1	2		
				P(x)	$\frac{144}{169}$	$\frac{24}{169}$	$\frac{1}{169}$		
Q30.	$\frac{dy}{dx}=(x)^{sinx}\left(\frac{sinx}{x}+cosx\ logsinx\right)+$ $(sinx)^x(xcotx+logsinx)$	**Q32**	(i) $(-\infty,-2)\cup(1{,}3)$ (ii) $(.-2{,}1)\cup(3,\infty)$						
Q33.	$\boldsymbol{x\ log(logx)-\frac{x}{logx}+C}$	**Q34**	Area=$2\sqrt{2}+9sin^{-1}\left(\frac{1}{3}\right)$	$\frac{3}{\pi}+\frac{1}{\pi^2}$					
Q35.	$sin\left(\frac{y}{x}\right)=log\lvert cx\rvert$								
Q36.	$\frac{8\sqrt{29}}{29}$	**OR**	7x-5y+4z-8=0						
Q37.	0.95	**OR**	$\frac{56}{83}$						
Q38.	Min Z= 2300	**OR**	200 desktop model and 50 portable model. Maximum profit = ₹ 1150000						

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