INDIAN SCHOOL AL WADI AL KABIR
Model Question Paper – XII (Set 3)
Mathematics 2020 - 2021

General Instructions:

1. *This question paper contains two parts A and B. Each part is compulsory. Part A has Objective Type Questions carries 24 marks and Part B has Descriptive Type Questions carries 56 marks.*
2. *Both Part A and Part B have choices.*

Part – A:

1. *It consists of two sections - I and II.*
2. *Section I comprises of 16 very short answer type questions.*
3. *Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.*

Part – B:

1. *It consists of three sections - III, IV and V.*
2. *Section III comprises of 10 questions of 2 marks each.*
3. *Section IV comprises of 7 questions of 3 marks each.*
4. *Section V comprises of 3 questions of 5 marks each.*
5. *Internal choice is provided in 3 questions of Section – III, 2 questions of Section - IV and 3 questions of Section - V. You have to attempt only one of the alternatives in all such questions.*

Part – A

Section – I

Question numbers 1 to 16 are very short answer type questions.

1. The value of $\cos^{-1}\left(\cos\frac{14\pi}{3}\right)$ is ....

2. Define onto function.

OR

Show that the function f in $A = R - \left\{\frac{2}{3}\right\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is onto.

3. Let $R = \{(x, x^2) : x$ is a prime number less than 15$\}$ be a relation. Find the range of R.

4. If A is a skew-symmetric matrix, then find A^2.

5. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then $A + A' = I$, then find the value of α.

OR

If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ -4 & 7 \end{bmatrix}$, then find $3A - B$.

6. Let A and B be two given mutually exclusive events. Then find $P(A/B)$.

7. If A is a matrix of order 3×3, then find the number of minors in determinant of A.

OR

If A is symmetric matrix, then explain $B'AB$.

8. Find the distance of the point (α, β, γ) from y-axis.

OR

A feasible solution of LPP is ....

9. The random variable X has a probability distribution $P(X)$ of the following form, where 'k' is some number.

$$P(X = x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & \text{otherwise} \end{cases}$$

Then find the value of 'k'.

10. What is the distance (in units) between the two planes $3x + 5y + 7z = 3$ and $9x + 15y + 21z = 9$?

11. If vectors $\hat{i}+\hat{j}-3\hat{k}$, $2\hat{i}+\hat{j}-\lambda\hat{k}$, and $5\hat{i}+2\hat{j}+3\hat{k}$ are coplanar, then find the value of λ.

12. What is the magnitude of projection $\left(2\hat{i}-\hat{j}+\hat{k}\right)$ on $\left(\hat{i}-2\hat{j}+2\hat{k}\right)$.

13. Find the value of $(\hat{k}\times\hat{j}).\hat{i}+\hat{j}.\hat{k}$.

14. Find the value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x \, dx$.

15. Find the differential coefficient of $f(x) = \sin 3x$.

16. Evaluate : $\int_1^2 \frac{x^3-1}{x^2} dx$.

OR

Evaluate : $\int_e^{e^2} \frac{dx}{x\log x}$

Section – II

Both the case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18. Each question carries 1 Mark

17 64 is the sum of two parts (numbers). Let A and B be the two numbers, then A + B = 64.
Now, let suppose y is the sum of cubes of both parts (numbers).
i.e., $y = A^3 + B^3$
Based on the above information answer the following questions :

i) What is the value of y in terms of A ?
(a) $A^3 + (64 - A)^3$ **(b)** $A^3 + (64 - A^3)$ **(c)** $A^3 + (A^3 - 64)$ **(d)** $A^3 + (64 + A)^3$

ii) Find the value of $\frac{dy}{dA}$?
(a) 3(128A + 4096) **(b)** 3(128A – 4096) **(c)** 3(4086 – 127A) **(d)** 3(127A – 4086)

iii) Find the value of A when $\frac{dy}{dA} = 0$.
(a) 32 **(b)** 48 **(c)** 50 **(d)** 30

iv) Find the value of $\frac{d^2y}{dA^2}$.
(a) 382 **(b)** 380 **(c)** 384 **(d)** 386

v) Find the value of B.
(a) 32 **(b)** 16 **(c)** 14 **(d)** 34

18. A company has two plants to manufacture bicycles. The first plant manufactures 60% of the bicycles and the second plant, 40%. Also, 80% of the bicycles are rated of standard quality at the first plant and 90% of standard quality at the second plant. A bicycle is picked up at random and found to be standard quality.
Let E_1 and E_2 be the events of choosing a bicycle from the first plant and the second plant respectively. Then, E be the event of choosing a bicycle of standard quality.
Based on the above information answer the following questions :

i) Find the value of $P(E_1)$.
(a) $\frac{2}{5}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{3}{7}$

ii) Find the value of $P(E_2)$.
(a) $\frac{2}{5}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{3}{7}$

iii) $P(E/E_1) = ?$
(a) $\frac{3}{5}$ **(b)** $\frac{2}{5}$ **(c)** $\frac{4}{5}$ **(d)** $\frac{9}{10}$

iv) $P(E/E_2) = ?$

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$ (c) $\frac{4}{5}$ (d) $\frac{9}{10}$

v) Find $P(E_2/E)$?

(a) $\frac{3}{7}$ (b) $\frac{4}{5}$ (c) $\frac{9}{10}$ (d) $\frac{3}{5}$

Part – B

Section – III

Question numbers 19 to 28 carry 2 marks each.

19. Let $f: N \to Y$ be a function defined as $f(x) = 4x + 3$, where $Y = \{y \in N ; y = 4x + 3$, for some $x \in N\}$. Show that f is one-one.

20. If $\begin{bmatrix} 2x & 4 \end{bmatrix}\begin{bmatrix} x \\ -8 \end{bmatrix} = 0$, find the positive value of x.

21. If $(x^2 + y^2)^2 = xy$, find $\frac{dy}{dx}$.

OR

If $x = a\sec^3\theta$, $y = a\tan^3\theta$, find $\frac{d^2y}{dx^2}$

22. Find the equation of the tangent to the curve $16x^2 + 9y^2 = 145$ at the point (x_1, y_1), where $x_1 = 2$ and $y_1 > 0$.

23. If $\vec{a}, \vec{b}$ are two vectors such that $|\vec{a}+\vec{b}| = |\vec{a}|$, then prove that $2\vec{a}+\vec{b}$ is perpendicular to $\vec{b}$.

OR

Using vectors, find the area of triangle ABC, with vertices $A(1, 2, 3)$, $B(2, -1, 5)$ and $C(4, 5, -1)$.

24. The equation of a line is

$5x - 3 = 15y + 7 = 3 - 10z$. Write the direction cosines of the line.

25. If $P(A) = \frac{2}{5}, P(B) = \frac{3}{10}$ and $P(A \cap B) = \frac{1}{5}$. then find $P(A'/B').P(B'/A')$.

26. Write the integrating factor of the differential equations $\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}}$.

27. Find the interval in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly increasing.

28. Evaluate : $\int (\cot x - \operatorname{cosec}^2 x) e^x \, dx$.

OR

Find : $\int \frac{3x}{3x-1} \, dx$

Section – IV

Question numbers 29 to 35 carry 3 marks each.

29. Let $A = \{1, 2, 3, ..., 9\}$ and R be the relation in $A \times A$ defined by $(a, b)\ R\ (c, d)$ if $a + d = b + c$ for $(a, b), (c, d)$ in $A \times A$. Prove that R is an equivalence relation. Also obtain the equivalence class $[(2, 5)]$.

30. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x - y)$, then prove that $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

OR

If $(a + bx)\, e^{y/x} = x$, then prove that $x^3 \frac{d^2y}{dx^2} = \left(x\frac{dy}{dx} - y\right)^2$

31. Find the particular solution of the differential equation $\frac{dy}{dx} = \frac{x(2\log x + 1)}{\sin y + y\cos y}$ given that $y = \frac{\pi}{2}$ where, $x = 1$.

OR

Solve the differential equation : $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$

32. Find : $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} \, dx$

33. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$. Also show that the maximum volume of the cone is $\frac{8}{27}$ of the volume of the sphere.

34. Find : $\int \frac{2x}{(x^2+1)(x^4+4)} \, dx$

35. Evaluate : $\int_1^4 \{|x-1| + |x-2| + |x-4|\} \, dx$

Section – V

Question numbers 36 to 38 carry 5 marks each.

36. Using matrix method, solve the system of equations

$$3x + 2y - 2z = 3,$$
$$x + 2y + 3z = 6,$$
$$2x - y + z = 2.$$

OR

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the windows to admit maximum light through the whole opening.

37. If lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1}$ intersect, then find the value of k and hence find the equation of plane containing these lines.

OR

Prove that the line through $A(0, -1, -1)$ and $B\ (4, 5, 1)$ intersects the line through $C\ (3, 9, 4)$ and $D\ (-4, 4, 4)$.

38. Solve the following linear programming problem graphically.

Maximize $Z = \frac{x}{10} + \frac{9y}{100}$

Subject to constraints

$$x + y \le 50{,}000$$
$$x \ge 20{,}000$$
$$y \ge 10{,}000$$

OR

Consider the experiment of tossing a coin. If the coin shows head, toss is done again, but if it shows tail, then throw a die. Find the conditional probability of the events that 'the die shows a number greater than 4', given that 'there is atleast one tail'.