INDIAN SCHOOL AL WADI AL KABIR
Model Question Paper – XII (Set 4)
Mathematics 2020 - 2021

General Instructions:

1. This question paper contains two parts A and B. Each part is compulsory. Part A has Objective Type Questions carries 24 marks and Part B has Descriptive Type Questions carries 56 marks.
2. Both Part A and Part B have choices.

Part – A:

1. It consists of two sections - I and II.
2. Section I comprises of 16 very short answer type questions.
3. Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

1. It consists of three sections - III, IV and V.
2. Section III comprises of 10 questions of 2 marks each.
3. Section IV comprises of 7 questions of 3 marks each.
4. Section V comprises of 3 questions of 5 marks each.
5. Internal choice is provided in 3 questions of Section – III, 2 questions of Section - IV and 3 questions of Section - V. You have to attempt only one of the alternatives in all such questions.

Part – A

Section – I

Question numbers 1 to 16 are very short answer type questions.

1. Find the value of $\sin^{-1}\left[\cos\left(\frac{33\pi}{5}\right)\right]$.

2. Let us define a relation R in R as aRb if $a \geq b$. Then what do you say about R ?

3. Let $A = \{1, 2, 3\}$ and consider the relation $R = (1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$. Then define R.

4. If A is matrix of order $m \times n$ and B is a matrix such that AB' and $B'A$ are both defined, then find the order of matrix B.

5. If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then find the value of x.

6. If $A = \begin{bmatrix} 2 & 3 \\ 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ -5 & -1 \end{bmatrix}$, then find $A - B$.

7. Write the value of $\vec{p}$ for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel vectors.

OR

Find the differential coefficient of $\operatorname{cosec}^{-1} x$.

8. A line passes through the point with position vector $2\hat{i} - 3\hat{j} + 4\hat{k}$ and makes angles 60°, 120°, and 45° with *x*, *y* and *z*-axis respectively. Find the equation of the line in the Cartesian form.

9. Write the vector equation of a line passing though the point (2, – 3, 1) and parallel to the line $\frac{x-4}{1} = \frac{y-2}{3} = \frac{z+1}{-2}$.

10. Find the value of $(\hat{k} \times \hat{j}).\hat{i} + \hat{j}.\hat{k} + 3$.

11. Find the vector in the direction of $\vec{a} = \hat{i} - 8\hat{j}$ that has magnitude 5 units.

12. Three balls are drawn from a bag containing 2 red and 5 black balls, if the random variable *x* represents the number of red balls drawn, then find the value of *x*.

OR

The probability distribution of the discrete variable X is given as : The value of *k* is ....

X	2	3	4	5
***P*(X)**	$\frac{5}{k}$	$\frac{7}{k}$	$\frac{9}{k}$	$\frac{11}{k}$

13. Two dice are thrown. If it is known that the sum of numbers on the dice was less than 6, then find the probability of getting a sum 3.

14. Evaluate : $\int \cos^{-1}(\sin x)\,dx$.

OR

Evaluate : $\int_0^1 e^{x^2} x\,dx$.

15. **16.** Find the intervals in which $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is strictly increasing.

OR

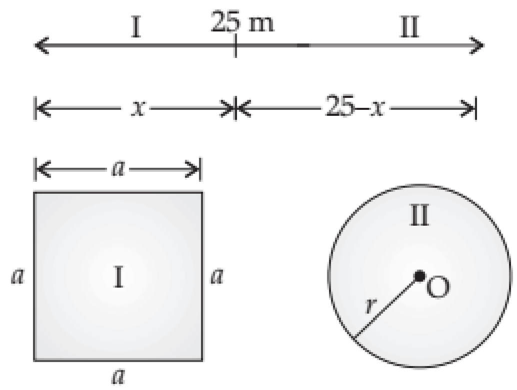
The objective function for a L.P. model is $3x_1 + 2x_2$, if $x_1 = 20$ and $x_2 = 30$, what is the value of the objective function ?

16. Find the derivative of $\sin^{-1} x$ w.r.t. *x*.

Section – II

Both the case study based questions are compulsory. Attempt any 4 sub parts from each question 17 and 18. Each question carries 1 Mark

17 A wire of length 25 m is to be cut into two pieces. One of the wires is to be made into a square and the other into a circle.



Based on the above information answer the following questions :

i) **(i)** Find the radius of circle (r).

(a) $\frac{25-x}{2\pi}$ **(b)** $\frac{25+x}{2\pi}$ **(c)** $\frac{2\pi}{25-x}$ **(d)** $\frac{2\pi}{25+x}$

ii) Find the combined area (A) of square and circle.

(a) $\frac{x^2}{16}+\frac{(x-25)^2}{4\pi}$ **(b)** $\frac{x^2}{16}+\frac{(25-x)^2}{4\pi}$ **(c)** $\frac{x^2}{4}+\frac{(x-25)^2}{4\pi}$ **(d)** $\frac{x^2}{4}+\frac{(25-x)^2}{4\pi}$

iii) Find the value of $\frac{dA}{dx}$.

(a) $\frac{(\pi+4)x-100}{8}$ **(b)** $\frac{(\pi+4)x-100}{\pi}$ **(c)** $\frac{(\pi+4)x-100}{8\pi}$ **(d)** $\frac{(\pi+4)-100}{8\pi}$

iv) Find $\frac{d^2A}{dx^2}$.

(a) $\frac{\pi-4}{8\pi}$ **(b)** $\frac{4-\pi}{8\pi}$ **(c)** $\frac{\pi+4}{\pi}$ **(d)** $\frac{\pi+4}{8\pi}$

v) Find the value of x, when $\frac{dA}{dx}=0$.

(a) $\frac{25\pi}{\pi+4}$ **(b)** $\frac{100}{\pi+4}$ **(c)** $\frac{25}{\pi+4}$ **(d)** $\frac{100\pi}{\pi+4}$

18. A man is known to speak the truth 3 out of 4 times. He throws a die and reports that it is a six.
Let, E_1 = event of getting a six.
E_2 = event of not getting a six
E = event that the man reports that it is a six.

i) **(i)** Find the value of $P(E_1)$.

(a) $\frac{5}{6}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{4}{5}$

ii) **(ii)** $P(E_2) = ?$

(a) $\frac{5}{6}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{4}{5}$

iii) **(iii)** Find the value of $P(E/E_1)$.

(a) $\frac{1}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{3}{4}$

iv) **(iv)** $P(E/E_2) = ?$

(a) $\frac{1}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{3}{4}$

v) **(v)** Find the value of $P(E_1/E)$.

(a) $\frac{1}{6}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{3}{4}$

Part – B

Section – III

Question numbers 19 to 28 carry 2 marks each.

19. Let $A = R - \{2\}$, $B = R - \{1\}$. If $f : A \to B$ is a function defined by $f(x) = \left(\frac{x-1}{x-2}\right)$, show that f is onto.

20. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$.

OR

Find matrix X so that $X\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$.

21. Evaluate : $\int_0^{\pi/2} e^x(\sin x - \cos x)\,dx$.

22. Find the particular solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0$ given that $y = 1$ when $x = 0$.

23. If value of λ and μ if $(\hat{i}+3\hat{j}+9\hat{k})\times(3\hat{i}-\lambda\hat{j}+\mu\hat{k})=0$

OR

If $\vec{a}$ and $\vec{b}$ are two unit vectors such that $\vec{a}+\vec{b}$ is also a unit vector, then find the angle between $\vec{a}$ and $\vec{b}$.

24. In an entrance test there are multiple choice questions. There are four possible answers to each question of which one is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then find the probability that he was guessing.

25. Show that the points *A*, *B*, *C* with position vectors $2\hat{i}-\hat{j}+\hat{k}$, $\hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$ respectively, are the vertices of a right-angled triangle.

26. Find the solution of the differential equation $x\frac{dy}{dx}=\frac{y}{1+\log x}$.

OR

Find the integrating factor of differential equation $\cos x\frac{dy}{dx}+y\sin x=1$.

27. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to $2x - y + 9 = 0$.

28. Write the sum of the order and degree of the differential equation $1+\left(\frac{dy}{dx}\right)^4=7\left(\frac{d^2y}{dx^2}\right)^3$

Section – IV

Question numbers 29 to 35 carry 3 marks each.

29. Find the equations of the normal to the curve $y = 4x^3 - 3x + 5$ which are perpendicular to the line $9x - y + 5 = 0$.

OR

Prove that $y = \frac{4\sin\theta}{2+\cos\theta}-\theta$ is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$.

30. Consider $f : R-\left\{-\frac{4}{3}\right\}\to R-\left\{\frac{4}{3}\right\}$ given by $f(x)=\frac{4x+3}{3x+4}$. Show that f is bijective.

31. A point on the hypotenuse of a right triangle is at distance 'a' and 'b' from the sides of the triangle. Show that the minimum length of the hypotenuse is $(a^{2/3} + b^{2/3})^{2/3}$.

OR

Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

32. Find : $\int \frac{\sec x}{1 + \operatorname{cosec} x}\, dx$.

33. If $x = a(\cos 2\theta + 2\theta \sin 2\theta)$ and $y = a(\sin 2\theta - 2\theta \cos 2\theta)$, find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{8}$.

34. Solve the differential equation $(\tan^{-1} x - y)dx = (1 + x^2)\, dy$

35. Find the particular solution of the differential equation :

$x\frac{dy}{dx} - y + x \operatorname{cosec} \frac{y}{x} = 0$; given that $y = 0$, when $x = 1$

Section – V

Question numbers 36 to 38 carry 5 marks each.

36. If the lines $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{2} = \frac{z-6}{-5}$ are perpendicular, find the value of λ. Hence find whether the lines are intersecting or not.

OR

If $\hat{a}$ and $\hat{b}$ are unit vectors inclined an angle θ, then prove that

$$\tan \frac{\theta}{2} = \frac{|\vec{a} - \vec{b}|}{|\vec{a} + \vec{b}|}.$$

37. Solve the following L.P.P. graphically :

Maximize $Z = 40x + 50y$ Subject to constraints : $3x + y \leq 9$ $x + 2y \leq 8$ $x, y \geq 0$

OR

Let X denote the number of colleges where you will apply after your results and $P(X = x)$ denotes your probability of getting admission is x number of colleges. It is given that :

$$P(X = x) = \begin{cases} kx & , \text{ if } x = 0 \text{ or } 1 \\ 2kx & , \text{ if } x = 2 \\ k(5-x) & , \text{ if } x = 3 \text{ or } 4 \\ 0 & , \text{ if } x > 4 \end{cases}$$

38. A trust fund has ₹ 35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bond pays 10% interest per annum which will be given to an N.G.O. (Cancer Aid Society). Use matrix multiplication, determine how to divide ₹ 35,000 among two types of bonds if the trust fund obtains an annual total interest of ₹ 3,200.

OR

To promote the making of toilets for women, as organization tried to generate awareness through (i) house calls (ii) letters and (iii) announcements. The cost for each mode per attempt is given below :

(i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40

The number of attempts made in three villages X, Y and Z and given below :

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for the three villages separately, using matrices.